Notes on using the material in the lectures outlined below.

There are three hour long lectures, each lecture has an accompanying handout for the students. The student handout does not contain all the material given in this document, there are gaps that the student is required to fill in as the material is covered. There is a PowerPoint presentation containing images of the balancing scales from lecture 1, although these images could be drawn up on the board if preferred.

The student handouts contain a set of homework exercises at the end. The exercises at the end of handout two and three (Exercises 2.3 and 3.1) require additional questions to be chosen from discipline relevant formulae. Ideally the lecturer teaching the subject can choose these formulae and add them into the MS word document before distributing the handout to the students.

In this document, the symbol indicates a concept question to be answered using the Peer Instruction method is usually as described below (any deviations are explained in the notes for lecturers):

- The question is usually a multiple choice question but on occasion requires a written explanation.
- Students are asked to consider the question briefly and input their answers using Poll Everywhere.
- Class results are examined then students discuss the question in groups for a maximum of five minutes (see times indicated for each question) after which they vote again and the results examined. Hopefully the correct idea has spread.
- Some concept questions are solely for discussion but others need to be followed by a summary of key findings. Where the symbol appears, the lecturer summarises the key concept or identifies the correct answer clearly on the board, students must copy this down into their version of the notes.

Lecture 1

<u>Objectives</u>

- demonstrate that many students in the class cannot rearrange an equation correctly
- concept of equality and balancing scales
- why we need to rearrange formulae
- inverse operations
- common misconceptions

NOTE The following question is to test the students' understanding of the topic before teaching. It is hoped that students who cannot answer the question correctly will realise that they need to engage with the material. There is no need to go through the steps of the solution, just provide the final answer.

Rearrange the equation below to make L the subject.

$$R = \sqrt{\frac{VL + PR}{L}}$$

Equality, balancing scales and equivalent equations

An **equation** says that a pair of quantities are equal. It helps to think of a weighing scales in balance and of both sides of the equations 'weighing' the same amount.

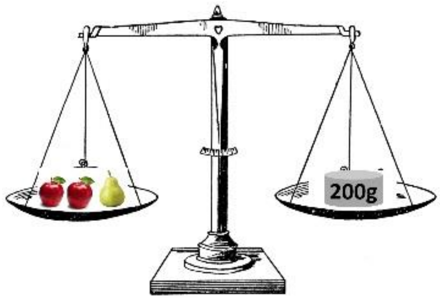
$$2 + 7 = 5 + 4$$

Add 3 to **both sides**

$$2 + 7 + \mathbf{3} = 5 + 4 \mathbf{+3}$$

Halve the amount on **both sides**

$$\frac{2+7+3}{2} = \frac{12}{2}$$



$$2a + p = 200$$

Double the amount (weight) on **both sides**

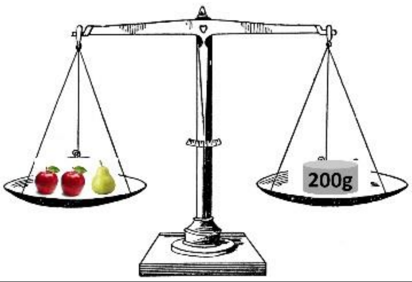
$$\mathbf{2} \times (2a + p) = \mathbf{2} \times 200$$

$$4a + 2p = 400$$

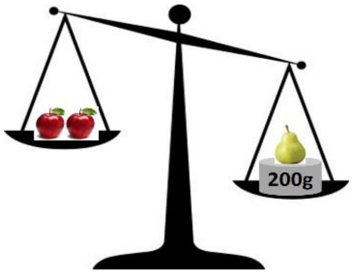
Add 50 to **both sides**

$$4a + 2p \mathbf{+ 50} = 400 \mathbf{+ 50}$$

We must **preserve the equality** thus keep the scales in **balance** at all times. If we **do the same thing** (apply the same operation) **to both sides** of the equation we keep the scales in balance, and create another (equivalent) equation.



$$2a + p = 200$$



What happened here?!!

$$2a = 200 + p$$

Poll Everywhere Q1.1

Can you explain why the scales are no longer balanced?

Key concept:

NOTE The question above is to be treated as a concept question (using the Poll Everywhere link – labelled Lecture 1 Question 1.1) Here students provide an explanation, their answers will appear onscreen as they are sent. There is no voting, just a written explanation followed by a two minute discussion. After discussion, the lecturer summarises the key concept outlined below.

Key concept: We cannot move things randomly across equations. In fact, the only action permitted is to do the same thing to both sides thus keeping the scales and the equation in balance.

NOTE The two questions below are concept questions but do not require the use of Poll Everywhere, just ask the students to tick the boxes and then discuss each option with their peers (two minutes per question). After the peer discussion, the lecturer discusses each option with the class and gives the correct answers. For question 3, please emphasise that the same operations have been done to both sides of the equation.

Poll Everywhere Q1.2

Which of the following are equations?

Tick as many boxes as apply.

- ☐ x^2
- ☐ $x^2 - 5x + 6 = 0$
- ☐ $x^2 = 2x$
- ☐ $x = 2b$
- ☐ $x + 5$
- ☐ $T = 2\pi\sqrt{\frac{l}{g}}$

Poll Everywhere Q1.3

If $3m = 5$ which of the following are true? Explain why.

Tick as many boxes as apply.

- ☐ $6m = 10$
- ☐ $9m^2 = 25$
- ☐ $3m - 2 = 3$
- ☐ $\sqrt{3m} = \sqrt{5}$
- ☐ $3m^2 = 25$
- ☐ $m = \frac{5}{3}$

Motivation

NOTE: Use example below or your own or a formula relevant to the students' discipline (see Departmental Formulae Inventory).

The relationship between temperature measured in degrees Fahrenheit ($°F$) and temperature in degrees Celsius ($°C$) is described by the equation below

$$F = \frac{9}{5}C + 32$$

where F represents the temperature in degrees Fahrenheit and C represents the temperature in degrees Celsius. If we have measured a temperature in $°F$ but need to convert it to $°C$ then we will need to rearrange the equation to make C the subject (note that a variable is the subject of the equation if it appears once, by itself, on one side of the equation)

Mathematical Inverses: operations that **undo** each other

To rearrange an equation correctly, it is necessary to understand the concept of an inverse function.

Operation	**Inverse**	**Example**
Adding	Subtracting	$a + 2 = b$ $a + 2 \mathbf{- 2} = b \mathbf{- 2}$ $a = b - 2$
Subtracting	Adding	$a - 2 = b$ $a - 2 \mathbf{+ 2} = b \mathbf{+ 2}$ $a = b + 2$
Multiplying	Dividing	$5 \times a = b$ $\frac{5 \times a}{\mathbf{5}} = \frac{b}{\mathbf{5}}$ $a = \frac{b}{5}$
Dividing	Multiplying	$\frac{a}{4} = b$ $\frac{a}{4} \mathbf{\times 4} = b \mathbf{\times 4}$ $a = 4b$

× (-1)	× (-1) ÷ (-1) is equivalent to × (-1)	$-a = b$ $-a \times (-1) = b \times (-1)$ $a = -b$
Squaring	Square root	$a^2 = b$ $\sqrt{a^2} = \sqrt{b}$ $a = \pm\sqrt{b}$
Square root	Squaring	$\sqrt{a} = b$ $(\sqrt{a})^2 = (b)^2$ $a = b^2$

Exercises 1.1

For each of the equations below, identify which operation is being applied to x and write down the corresponding inverse operation. In the last column you can write down the solution to the equation.

Equation	**Operation**	**Inverse**	**Solution**
$2x = 3$	$\times 2$	$\div 2$	$x = \frac{3}{2}$
$x + 2 = 3$			
$x^2 = 5$			
$x - a = 5$			
$\frac{x}{6} = b$			
$\sqrt{x} = 3$			

Discussion of common misconceptions

NOTE

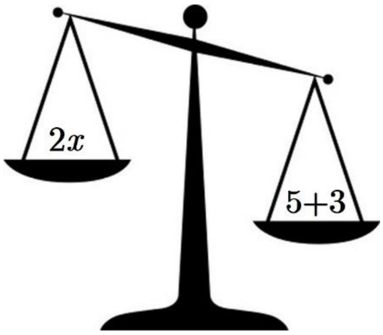
To avoid errors copied into students' notes ask the students to put their pens down discussing the following. The three questions below are concept questions (using the Poll Everywhere link labelled Lecture 1 Question 1.4, Question 1.5 and Question 1.6). For each question, first the students vote, then discuss the question amongst themselves for two minutes then vote again. After the second round of voting, the lecturer gives the correct answers and discusses with the students why it is correct.

Concept question 1.4

Common error due to misconception of **'moving' terms**

Suppose that we would like to rearrange the equation $2x + 3 = 5$ to make x the subject. Is the step shown below correct?

$$2x + 3 = 5$$
$$2x = 5 + 3$$



Concept question 1.5

Suppose that we would like to rearrange the equation $3x = 6$ to make x the subject. Is the step shown below correct?

$$3x = 6$$
$$x = \frac{6}{-3}$$

Concept question 1.6

What went wrong here? Ask the students to suggest the correct action.

$$8x = 4$$
$$x = \frac{8}{4}$$

Lecture 2

Objectives

- review of algebra prerequisites of transposition
- main approach to transposition with some worked examples
- discussion of general guidelines
- addressing the difficulty of 'where to start'

NOTE

Explain the first three laws in class with one example worked out by the lecturer and a matching one by students.

Rearranging equations: know right from wrong

True laws always apply	Why?	Exercise
$ax + ay = a(x+y)$	$2\,pears + 3\,pears = 5\,pears$ $2\mathbf{p} + 3\mathbf{p} = \mathbf{p}(2+3) = 5p$ $\mathbf{p}x + \mathbf{p}y = \mathbf{p}(x+y)$	$mt + ms =$
$\frac{a \cdot x}{b \cdot x} = \frac{a}{b}$	$\frac{4\mathbf{q}}{5\mathbf{q}} = \frac{4}{5} \times \frac{\mathbf{q}}{\mathbf{q}} = \frac{4}{5}$ $\frac{\mathbf{z} \cdot c}{d \cdot \mathbf{z}} = \frac{c}{d}$	$\frac{2k}{7k} =$ $\frac{2+k}{7k} =$ $\frac{a \cdot p}{p \cdot k} =$
$\frac{(a \cdot p + b \cdot p)}{bp} = a + b$	$\frac{\mathbf{3}s + \mathbf{3}t}{\mathbf{3}} = \frac{\mathbf{3}(s+t)}{\mathbf{3}} = s + t$	$\frac{5x + 5y}{5} =$ $\frac{5a+2p}{5} =$ $\frac{a \cdot x + a \cdot y}{a} =$
$(c+d)^2 = c^2 + 2cd + d^2$	Multiply out and simplify to see this for yourself $(c+d)^2 = (c+d)(c+d) =$	
$(5z)^2 = 25z^2$	$(5z)^2 = \mathbf{5}z \times \mathbf{5}\,z =$ $= \mathbf{5} \times \mathbf{5} \times z \times z == 5^2 z^2$	$(3n)^2 =$

Some definitions

Terms are the entities separated by addition or subtraction: $\underset{term}{2} \underset{term}{-3x^2} \underset{term}{+4a}$

A product is made up of **factors**: $\underset{factor}{4} \times \underset{factor}{7}$, $3pq = \underset{factor}{3} \times \underset{factor}{p} \times \underset{factor}{q}$

If the **same factor** is present in **every term** we can factor it out (put outside a bracket):

$$4a - 2 + 6p = \mathbf{2} \times 2a - \mathbf{2} \times 1 + \mathbf{2} \times 3p = \mathbf{2}(2a - 1 + 3p)$$

NOTE: To avoid errors copied into students' notes ask the students to put their pens down before discussing the following. The four questions below are concept questions (using the Poll Everywhere link labelled Lecture 2 Question 2.1, Question 2.2, Question 2.3 and Question 2.4). For each question, first the students vote, then discuss the question amongst themselves for three minutes then vote again. After the second round of voting, the lecturer gives the correct answers.

Poll Everywhere Q2.1

Which of the following equations are correct? Justify your choice.

(a) $\frac{a+3p}{p} = a + 3$ (b) $\frac{ap+3p}{p} = a + 3$

Key concept:

Equation (b) is correct. Where possible, we simplify fractions by dividing both numerator and denominator by the same quantity. Before simplifying, see whether a quantity is a factor in each term, factor it out, then consider simplifying. If you cannot factor something out, you cannot simplify.

$$\frac{ap + 3p}{p} = \frac{p(a + 3)}{p} = a + 3$$

Poll Everywhere Q2.2

Given the equation $x + 3 = \frac{\pi}{2}$

and after **doubling both sides** we obtain $2x + 3 = \pi$ True or false?

Key concept: False, when multiplying an equation by something you must multiply **each term** on both sides by that entity. Doubling both sides of the equation would result in $2x + 6 = \pi$.

Poll Everywhere Q2.3

After squaring both sides of the equation $\sqrt{x} = q + 4$ we obtain:

(a) $x = q^2 + 16$? (b) $x = q^2 + 8q + 16$?

Key concept: Square of a sum is not equal to the sum of squares. Squaring the right hand side of the equation gives $(q + 4)^2 = (q + 4)(q + 4)$

Poll Everywhere Q2.4

$\sqrt{p^2 - r^2} = p - r$ True or false?

Key concept: In general $\sqrt{p^2 - r^2} \neq p - r$ this can be shown by choosing numbers for p and q and checking.

NOTE

The following exercises are for the students to do in class. After a few minutes, give and explain the correct answers on the board.

Exercises 2.1

1. Which of these is the solution to $\frac{x}{7} = 56$?

   (a) $x = 49$ (b) $x = 392$ (c) $x = 8$ (d) $x = 63$

2. Make p the subject of the equation $m = \sqrt{p + 9}$

   (a) $p = m^2 - 81$ (b) $p = m^2 - 9$

   (c) $p = \sqrt{m} - 3$ (d) $p = (m - 9)^2$

3. What does $\frac{3m+m^2}{m}$ simplify to?

   (a) $3 + m^2$ (b) $3m^2 + m^3$

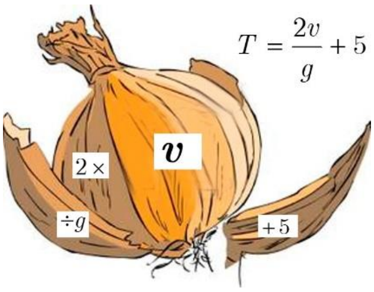
   (c) $3 + m$ (d) none of the above

Principles of rearranging formulae

- we can do anything to an equation as long as we **do the same thing to both sides**;
- we simplify/rearrange equations **step by step** by applying **inverse operations.**

More general guidelines

- Perhaps start by removing the entities that are furthest away from your subject. Work in small steps, removing one operation at a time, thus getting closer and closer to the subject.
- Think of peeling an onion, removing outer layers before getting to the core, as an analogy of getting to the subject in your formula.



- It is OK to do many small (but correct) steps. It is also OK to only think of the next small step instead of 'having to plan the entire route in detail from the start'.

Examples

NOTE Two examples to be worked out: (i) by the lecturer and (ii) by the students.

(i) Rearrange $T = \frac{2v}{g} + 5$ to make $\boldsymbol{v}$ the subject.

(ii) Rearrange $q = \frac{4t}{5} - 2$ to make $\boldsymbol{t}$ the subject.

Exercise 2.2: what's the first step towards isolating the variable?

Equation	Want to get rid off	Do to both sides	New equation
$4s - 7 = 10$			
$3 \cdot \sqrt{z^2 - 4} = 6$			
$\frac{2p + 5}{3} = 4$			
$\sqrt{x - 6} = 2$			

Homework 1 & 2: Homework/Tutorial questions (see student handout). Five additional questions to be chosen from discipline relevant formulae

Lecture 3

Objectives:

- Provide discipline relevant examples
- Special focus on fractions
- Factorising when the variable of interest appears more than once

NOTE

Give the Transposition Memento to the students at the start of the class so they can consult with it while doing exercises. Hard copies of Memento are available from Catherine Palmer or can be downloaded from here , printed in colour on A4 paper and folded in two to look like an oversized bookmark.

Variables represent physical quantities and are the names agreed by scientists within a specific discipline. These names aren't necessarily just one symbol (letter) but often are a combination of symbols, sometimes also involving subscripts. Just like name Mary involves more than one letter, change in temperature and concentration of chemical 1 are usually denoted (named) by ΔT and C_1.

Principles of rearranging formulae

Recall:

- we can do anything to an equation as long as we **do the same thing to both sides**;
- we simplify/rearrange equations **step by step** by applying **inverse operations**.
- Apply the same inverse operations to both sides of the equation.

NOTE

The following examples are to be answered on the board for students to copy down. The examples cover a range of formulae encountered across many disciplines in CIT. Ideally, any formulae that is not relevant to the class being taught can be replaced with an **equivalent** formulae relevant to the discipline. Depending on how the class is progressing, let the students try to rearrange formulae 3 and 4 by themselves first.

Examples

1. Rearrange $Q = mc\Delta T$ to make m the subject.
2. Rearrange $C_1V_1 = C_2V_2$ to make V_1 the subject.
3. Transpose the formula below to make to make m the subject.

$$\rho = \frac{m}{V}$$

4. Rearrange $P = RI^2$ to make I the subject.
5. Transpose the formula below to make to make g the subject.

$$\omega = \sqrt{\frac{g}{L}}$$

Factorising

Sometimes the variable of interest can appear more than once in an equation. Rearrange the equation to have **all terms with the variable** of interest **on one side**, then **factorise**.

Examples

NOTE

Two examples to be worked out: (i) by the lecturer and (ii) by the students.

(i) Rearrange $xy + z = x$ to make x the subject.

(ii) Rearrange $at - u = 5t$ to make t the subject.

Fractions

When rearranging equations that involve fractions, it can be helpful to 'get rid' of the fractions by multiplying both sides of the equation by the denominator(s). This is particularly helpful when the variable of interest is present in the denominator.

Examples

NOTE

Two examples to be worked out: (i) by the lecturer and (ii) by the students.

(i) Rearrange $\frac{1}{2+x} - 4 = 5$ to make x the subject.

(ii) Rearrange $\frac{2}{a-2} = 5$ to make a the subject.

Hint: If there are **several** fractions present in an equation, we can 'get rid' of all the denominators in one step by multiplying by the **product of the denominators**.

Examples

NOTE

Two examples to be worked out: (i) by the lecturer and (ii) by the students.

(i) Rearrange $\frac{1}{R} + \frac{1}{R_1} = \frac{1}{R_2}$ to make R the subject.

(ii) Rearrange $\frac{1}{a} + \frac{1}{b} = 5$ to make a the subject.

Homework 3 (NOTE 6 additional questions to be chosen from discipline relevant formulae and added in the Handout 3.)

1. Rearrange $\frac{M}{I} = \frac{E}{R}$ to make R the subject.
2. Rearrange $y = \frac{n\lambda L}{d}$ to make L the subject.
3. Rearrange $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ to make v the subject.